ELSEVIER

Journal of Computational and Applied Mathematics 124 (2000) ix–x

JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS

www.elsevier.nl/locate/cam

Preface

Numerical Analysis 2000
Vol. IV: Optimization and Nonlinear Equations

In one of the papers in this collection, the remark that "nothing at all takes place in the universe in which some rule of maximum or minimum does not appear" is attributed to no less an authority than Euler. Simplifying the syntax a little, we might paraphrase this as *Everything is an optimization problem*. While this might be something of an overstatement, the element of exaggeration is certainly reduced if we consider the extended form: *Everything is an optimization problem or a system of equations*. This observation, even if only partly true, stands as a fitting testimonial to the importance of the work covered by this volume.

Since the 1960s, much effort has gone into the development and application of numerical algorithms for solving problems in the two areas of optimization and systems of equations. As a result, many different ideas have been proposed for dealing efficiently with (for example) severe nonlinearities and/or very large numbers of variables. Libraries of powerful software now embody the most successful of these ideas, and one objective of this volume is to assist potential users in choosing appropriate software for the problems they need to solve. More generally, however, these collected review articles are intended to provide both researchers and practitioners with snapshots of the 'state-of-the-art' with regard to algorithms for particular classes of problem. These snapshots are meant to have the virtues of immediacy through the inclusion of very recent ideas, but they also have sufficient depth of field to show how ideas have developed and how today's research questions have grown out of previous solution attempts.

The most efficient methods for *local optimization*, both unconstrained and constrained, are still derived from the classical Newton approach. The papers in this collection describe many interesting variations, particularly with regard to the organization of the linear algebra involved. The popular quasi-Newton techniques avoid the need to calculate second derivatives, but gradient-based methods have received more attention than their direct search (function-values only) counterparts. This volume does, however, include an up-to-date account of available direct search techniques, which are required, for instance, for problems where function-values are subject to uncertainty.

As well as dealing in depth with the various classical, or neo-classical, approaches, the selection of papers on optimization in this volume ensures that newer ideas are also well represented. Thus the reader will find an account of the impact that ABS methods for linear systems are beginning to make upon the subject. The potential of interval arithmetic for dealing with the global optimization problem is also discussed, as are the emerging methods and software tools of automatic differentiation for supplying the derivative information needed by most optimization techniques. No volume on optimization would be considered to have the necessary breadth unless the vital topics of linear programming (to quote from one of the papers: "undoubtedly the optimization problem solved most

PII: S0377-0427(00)00416-7

frequently in practice") and its related areas were given due attention. As a consequence, papers on interior point methods (covering both linear and nonlinear problems), complementarity, and both integer and combinatorial optimization may all be found here. A state-of-the-art review of numerical methods in the field of optimal control is also included.

Solving *nonlinear algebraic systems of equations* is closely related to optimization. The two are not completely equivalent, however, and usually something is lost in the translation. Reformulating an optimization problem as a nonlinear system often introduces spurious stationary point solutions that are not local optima of the original problem. Conversely, formulating a nonlinear system of equations as minimizing some merit function often introduces local minima of the merit function that are not roots of the original nonlinear system. Nevertheless, the algorithms and conceptual approaches for optimization and nonlinear systems of equations are closely related, and a new algorithmic trick for one generally suggests an analogous trick for the other.

Algorithms for nonlinear equations can be roughly classified as *locally convergent* or *globally convergent*. The characterization is not perfect. A globally convergent method applied to a problem for which no global convergence theory exists may only converge locally. Often strong claims of global convergence are made for locally convergent methods (e.g., trust region), but the convergence is to a stationary point of some merit function, which is not necessarily a true *solution* to the nonlinear system of equations. The suggested classification is useful, because some approaches use local models and local theory, while others use the global behavior of the function. Local and global analysis are qualitatively very different, and from this perspective so are the algorithms based on these respective theories.

Locally convergent algorithms include Newton's method, modern quasi-Newton variants of Newton's method, and trust region methods. All of these approaches are well represented in this volume. Globally convergent algorithms include pattern search, continuation, homotopy, probability-one homotopy, and interval methods. The homotopy approach has both discrete (simplicial) and continuous variants. All of these (except for the relatively new, globally convergent, probability-one homotopy algorithms) are described in this volume by researchers with a lifetime of experience in the topics, often from a uniquely personal perspective.

Finally, the editors wish to place on record their deep appreciation of the assistance given by many referees in the preparation of this volume. Without their generous and continued co-operation, it is doubtful whether this volume would have appeared on schedule. To all of them, we offer our thanks. We also gladly acknowledge the guidance and assistance we have received from Luc Wuytack throughout the process of preparing this volume.

Layne T. Watson
Departments of Computer Science and Mathematics
Virginia Polytechnic Institute & State University
Blacksburg, VA 24061-0106, USA
Michael Bartholomew-Biggs
Numerical Optimisation Centre, University of Hertfordshire
Hatfield, Hertfordshire AL10 9AB, UK
John Ford
Department of Computer Science, University of Essex
Wivenhoe Park, Colchester, Essex CO4 3SQ, UK
E-mail address: fordj@essex.ac.uk